Spiral cylindrique avec courbes terminales en arc de cercle

Poids du spiral et anisochronisme en position verticale

Déformations planes

Caractéristiques du spiral **dextre**

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-9}$

Elinvar $\quad \rho_s = 8 \times 10^3\ kg\cdot m^{-3} \quad E = 1.7 \times 10^{11}\,Pa \quad G = 6.538 \times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654 \times 10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

$$r_s(\alpha) := R_0 \qquad s(\alpha) := R_0\cdot(\alpha - \pi) \qquad x_{0s}(\alpha) := R_0\cdot cos(\alpha) \qquad y_{0s}(\alpha) := R_0\cdot sin(\alpha)$$

Courbe terminale externe $\quad \beta := 121\cdot deg \quad \beta_0 := racine\left[\beta\cdot\left(\sqrt{2}\cdot sin(\beta) - 1\right) + sin(\beta)\cdot cos(\beta), \beta\right] \quad \beta_0 = 121.21\,deg$

$$\alpha_A := \pi \qquad r_t := \frac{R_0}{\sqrt{2}\cdot sin(\beta_0)} \qquad x_{0t}(\alpha_t) := -R_0 + r_t\cdot(1 + cos(\alpha_t)) \qquad y_{0t}(\alpha_t) := r_t\cdot sin(\alpha_t) \qquad l_t := r_t\cdot 2\cdot\beta_0$$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0 + \pi, 2\cdot\pi) \quad \alpha_B = 234\,deg$

$$x_{0t'}(\alpha_{t'}) := \left[R_0 + r_t\cdot(-1 + cos(\alpha_{t'}))\right]cos(\alpha_B) - r_t\cdot sin(\alpha_{t'})\cdot sin(\alpha_B)$$

$$y_{0t'}(\alpha_{t'}) := \left[R_0 + r_t\cdot(-1 + cos(\alpha_{t'}))\right]sin(\alpha_B) + r_t\cdot sin(\alpha_{t'})\cdot cos(\alpha_B) \qquad L_t := 2\cdot l_t + L$$

Position du piton $\quad \alpha_{tP} := \pi - 2\cdot\beta_0 \quad \alpha_{tP} = -62.426\,deg \quad x_P := x_{0t}(\alpha_{tP}) \quad y_P := y_{0t}(\alpha_{tP})$

$$z_P := x_P + i\cdot y_P \qquad r_P := |z_P| \qquad r_P = 3.811\,mm \qquad arg(z_P) = -74.047\,deg$$

Position du point d'attache à la virole $\quad r_V := r_P \quad \alpha_V(\theta) := Atan\left(x_{0t'}(2\cdot\beta_0), y_{0t'}(2\cdot\beta_0)\right) + \theta \quad \alpha_V(0) = 128.047\,deg$

$$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \qquad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$$

Amplitude stationnaire du balancier $\quad \theta_0 := 270\cdot deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha)$$

Première approximation de la déformée du spiral

Courbe terminale externe

$$\varphi_{0t}(\alpha_t) := \alpha_t + \frac{\pi}{2} \qquad z_{1t}(\theta, \alpha_t) := z_P + r_t\cdot\int_{\alpha_{tP}}^{\alpha_t} i\cdot exp(i\cdot\alpha'_t)\cdot exp\left[i\cdot\theta\cdot\frac{r_t}{L_t}\cdot(\alpha'_t - \alpha_{tP})\right] d\alpha'_t$$

$$z_{1t}(\theta, \alpha_t) := z_P + \frac{r_t\cdot L_t}{L_t + \theta\cdot r_t}\cdot\left(exp\left(-i\cdot\frac{-\alpha_t\cdot L_t - \theta\cdot r_t\cdot\alpha_t + \theta\cdot r_t\cdot\alpha_{tP}}{L_t}\right) - exp(i\cdot\alpha_{tP})\right)$$

Partie cylindrique

$$\varphi_0(\alpha) := \alpha + \frac{\pi}{2} \qquad \Delta z_{1s}(\theta,\alpha) := R_0 \cdot \int_{\pi}^{\alpha} i \cdot exp(i \cdot \alpha') \cdot exp\left(i \cdot \theta \cdot R_0 \cdot \frac{\alpha' - \pi}{L_t}\right) d\alpha'$$

$$\Delta z_{1s}(\theta,\alpha) := \frac{R_0 \cdot L_t}{L_t + \theta \cdot R_0} \cdot \left(exp\left(-i \cdot \frac{-\alpha \cdot L_t - \theta \cdot R_0 \cdot \alpha + \theta \cdot R_0 \cdot \pi}{L_t}\right) + 1\right) \qquad z_{1A}(\theta) := z_{1t}(\theta,\pi)$$

$$\Delta\varphi_{1A}(\theta) := \theta \cdot \frac{l_t}{L_t} \qquad \Delta\varphi_{1A}(\theta_0) = 13.346\,deg \qquad z_{1s}(\theta,\alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta,\alpha) \cdot e^{i \cdot \Delta\varphi_{1A}(\theta)}$$

Courbe terminale interne

$$\Delta z_{1t'}(\theta,\alpha_{t'}) := r_{t'} \cdot \int_{0}^{\alpha_{t'}} i \cdot exp(i \cdot \alpha'_{t'}) \cdot exp\left(i \cdot \theta \cdot \frac{r_t}{L_t} \cdot \alpha'_{t'}\right) d\alpha'_{t'}$$

$$\Delta z_{1t'}(\theta,\alpha_{t'}) := \frac{r_t \cdot L_t}{\theta \cdot r_t + L_t} \cdot \left(exp\left(i \cdot \alpha_{t'} \cdot \frac{\theta \cdot r_t + L_t}{L_t}\right) - 1\right) \qquad z_{1B}(\theta) := z_{1s}(\theta, \psi_0 + \pi) \qquad \alpha_B = 234\,deg$$

$$\alpha_{1B}(\theta) := Atan\big(Re(z_{1B}(\theta)), Im(z_{1B}(\theta))\big) \qquad z_{1t'}(\theta,\alpha_{t'}) := z_{1B}(\theta) + \Delta z_{1t'}(\theta,\alpha_{t'}) \cdot e^{i \cdot \alpha_{1B}(\theta)}$$

Première approximation du déplacement du centre de gravité

Contribution du spiral sans ses courbes terminales

$$s_s(\alpha) := R_0 \cdot (\alpha - \pi) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + i \cdot y_{0s}(\alpha) \qquad f_s(\theta,\alpha) := i \cdot \theta \cdot exp\left(i \cdot \theta \cdot \frac{s_s(\alpha)}{L_t}\right)$$

$$\boldsymbol{\Delta_s}(\theta) := \frac{R_0}{L_t} \cdot \int_{\pi}^{\pi + \psi_0} z_{0s}(\alpha) \cdot f_s(\theta,\alpha)\, d\alpha \qquad \boldsymbol{\Delta_s}(\theta_0) = 0.1 + 0.309i\,mm$$

Contribution de la courbe terminale externe

$$s_t(\alpha_t) := r_t \cdot (\alpha_t - \alpha_{tP}) \qquad z_{0t}(\alpha_t) := x_{0t}(\alpha_t) + i \cdot y_{0t}(\alpha_t) \qquad f_t(\theta,\alpha_t) := i \cdot \theta \cdot exp\left(i \cdot \theta \cdot \frac{s_t(\alpha_t)}{L_t}\right)$$

$$\boldsymbol{\Delta_t}(\theta) := \frac{r_t}{L_t} \cdot \int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t) \cdot f_t(\theta,\alpha_t)\, d\alpha_t \qquad \boldsymbol{\Delta_t}(\theta_0) = -0.286 - 0.059i\,mm$$

Contribution de la courbe terminale interne

$$\alpha_B = 234\,deg$$

$$s_{t'}(\alpha_{t'}) := r_{t'} \cdot \alpha_{t'} + L + l_t \qquad z_{0t'}(\alpha_{t'}) := x_{0t'}(\alpha_{t'}) + i \cdot y_{0t'}(\alpha_{t'}) \qquad f_{t'}(\theta,\alpha_{t'}) := i \cdot \theta \cdot exp\left(i \cdot \theta \cdot \frac{s_{t'}(\alpha_{t'})}{L_t}\right)$$

$$\boldsymbol{\Delta_{t'}}(\theta) := \frac{r_t}{L_t} \cdot \int_{0}^{2 \cdot \beta_0} z_{0t'}(\alpha_{t'}) \cdot f_{t'}(\theta,\alpha_{t'})\, d\alpha_{t'} \qquad \boldsymbol{\Delta_{t'}}(\theta_0) = 0.197 - 0.216i\,mm$$

Contribution du spiral entier

$$\boldsymbol{\Delta_1}(\theta) := \boldsymbol{\Delta_t}(\theta) + \boldsymbol{\Delta_s}(\theta) + \boldsymbol{\Delta_{t'}}(\theta) \qquad \boldsymbol{\Delta_1}(\theta_0) = 0.011 + 0.035i\,mm$$

$$u_1(\theta) := Re(\boldsymbol{\Delta_1}(\theta)) \qquad v_1(\theta) := Im(\boldsymbol{\Delta_1}(\theta)) \qquad u_1(\theta_0) = 0.011\,mm \qquad v_1(\theta_0) = 0.035\,mm$$

$$\xi_1(\theta) := \frac{d}{d\theta} v_1(\theta) - u_1(\theta) \qquad \eta_1(\theta) := -\frac{d}{d\theta} u_1(\theta) - v_1(\theta) \qquad \xi_1(\theta_0) = -2.884 \times 10^{-3}\,mm \qquad \eta_1(\theta_0) = -0.018\,mm$$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot R_0\, d\alpha + \int_{\alpha_{tP}}^{\pi} x_{0t}(\alpha_t)^2\cdot r_t\, d\alpha_t + \int_0^{2\cdot\beta_0} x_{0t'}(\alpha_{t'})^2\cdot r_t\, d\alpha_{t'}\right) \qquad p2_{0s} = 12.039\, mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} y_{0s}(\alpha)^2\cdot R_0\, d\alpha + \int_{\alpha_{tP}}^{\pi} y_{0t}(\alpha_t)^2\cdot r_t\, d\alpha_t + \int_0^{2\cdot\beta_0} y_{0t'}(\alpha_{t'})^2\cdot r_t\, d\alpha_{t'}\right) \qquad q2_{0s} = 12.105\, mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot R_0\, d\alpha + \int_{\alpha_{tP}}^{\pi} x_{0t}(\alpha_t)\cdot y_{0t}(\alpha_t)\cdot r_t\, d\alpha_t + \int_0^{2\cdot\beta_0} x_{0t'}(\alpha_{t'})\cdot y_{0t'}(\alpha_{t'})\cdot r_t\, d\alpha_{t'}\right)$$

$$k_{0s} = -0.046\, mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s}\end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} 9.979\times 10^{-6} \\ 3.121\times 10^{-5}\end{pmatrix} N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 3.277\times 10^{-5}\, N$$

Deuxième approximation de la déformée du spiral

$$R'_x(\theta) := \mathbf{R'}(\theta)_0 \qquad R'_y(\theta) := \mathbf{R'}(\theta)_1$$

$$x_{1t}(\theta,\alpha_t) := Re(z_{1t}(\theta,\alpha_t)) \qquad y_{1t}(\theta,\alpha_t) := Im(z_{1t}(\theta,\alpha_t))$$

$$x_{1s}(\theta,\alpha) := Re(z_{1s}(\theta,\alpha)) \qquad y_{1s}(\theta,\alpha) := Im(z_{1s}(\theta,\alpha))$$

$$x_{1t'}(\theta,\alpha_{t'}) := Re(z_{1t'}(\theta,\alpha_{t'})) \qquad y_{1t'}(\theta,\alpha_{t'}) := Im(z_{1t'}(\theta,\alpha_{t'}))$$

$$s\xi_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad s\xi_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')\cdot R_0\, d\alpha'$$

$$s\xi_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} x_{1t'}(\theta,\alpha'_{t'})\cdot r_t\, d\alpha'_{t'} \qquad \xi_1(\theta) := \frac{1}{L_t}\cdot\left(s\xi_{1t}(\theta,\pi) + s\xi_{1s}(\theta,\psi_0+\pi) + s\xi_{1t'}(\theta,2\cdot\beta_0)\right)$$

$$s\eta_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} y_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad s\eta_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} y_{1s}(\theta,\alpha')\cdot R_0\, d\alpha'$$

$$s\eta_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} y_{1t'}(\theta,\alpha'_{t'})\cdot r_t\, d\alpha'_{t'} \qquad \eta_1(\theta) := \frac{1}{L_t}\cdot\left(s\eta_{1t}(\theta,\pi) + s\eta_{1s}(\theta,\psi_0+\pi) + s\eta_{1t'}(\theta,2\cdot\beta_0)\right)$$

$$sp2_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sp2_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')^2\cdot R_0\, d\alpha'$$

$$sp2_{1t'}(\theta,\alpha_{t'}) := \int_0^{\alpha_{t'}} x_{1t'}(\theta,\alpha'_{t'})^2\cdot r_t\, d\alpha'_{t'}$$

$$sq2_{1t}(\theta,\alpha_t) := \int_{\alpha_{tP}}^{\alpha_t} y_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sq2_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} y_{1s}(\theta,\alpha')^2\cdot R_0\, d\alpha'$$

$$sq2_{1t'}\left(\theta,\alpha_{t'}\right) := \int_{0}^{\alpha_{t'}} y_{1t'}\left(\theta,\alpha'_{t'}\right)^2 \cdot r_t \, d\alpha'_{t'}$$

$$sk_{1t}\left(\theta,\alpha_t\right) := \int_{\alpha_{tP}}^{\alpha_t} x_{1t}\left(\theta,\alpha'_t\right)\cdot y_{1t}\left(\theta,\alpha'_t\right)\cdot r_t \, d\alpha'_t \qquad sk_{1s}(\theta,\alpha) := \int_{\pi}^{\alpha} x_{1s}(\theta,\alpha')\cdot y_{1s}(\theta,\alpha')\cdot R_0 \, d\alpha'$$

$$sk_{1t'}\left(\theta,\alpha_{t'}\right) := \int_{0}^{\alpha_{t'}} x_{1t'}\left(\theta,\alpha'_{t'}\right)\cdot y_{1t'}\left(\theta,\alpha'_{t'}\right)\cdot r_t \, d\alpha'_{t'}$$

$$\boldsymbol{S_t}\left(\theta,\alpha_t\right) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) + sq2_{1t}(\theta,\alpha_t) & \left(y_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t)\right) \\ x_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t) & -x_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) + sp2_{1t}(\theta,\alpha_t) \end{bmatrix}$$

$$\boldsymbol{S_s}(\theta,\alpha) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) + sq2_{1s}(\theta,\alpha) & \left(y_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha)\right) \\ x_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha) & -x_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) + sp2_{1s}(\theta,\alpha) \end{bmatrix}$$

$$\boldsymbol{S_{t'}}\left(\theta,\alpha_{t'}\right) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1t'}(\theta,\alpha_{t'})\cdot s\eta_{1t'}(\theta,\alpha_{t'}) + sq2_{1t'}(\theta,\alpha_{t'}) & \left(y_{1t'}(\theta,\alpha_{t'})\cdot s\xi_{1t'}(\theta,\alpha_{t'}) - sk_{1t'}(\theta,\alpha_{t'})\right) \\ x_{1t'}(\theta,\alpha_{t'})\cdot s\eta_{1t'}(\theta,\alpha_{t'}) - sk_{1t'}(\theta,\alpha_{t'}) & -x_{1t'}(\theta,\alpha_{t'})\cdot s\xi_{1t'}(\theta,\alpha_{t'}) + sp2_{1t'}(\theta,\alpha_{t'}) \end{bmatrix}$$

$$\boldsymbol{\Delta z_t}\left(\theta,\alpha_t\right) := \boldsymbol{S_t}\left(\theta,\alpha_t\right)\cdot\boldsymbol{R'}(\theta) \qquad \boldsymbol{\Delta z_s}(\theta,\alpha) := \boldsymbol{S_s}(\theta,\alpha)\cdot\boldsymbol{R'}(\theta) \qquad \boldsymbol{\Delta z_{t'}}\left(\theta,\alpha_t\right) := \boldsymbol{S_{t'}}\left(\theta,\alpha_t\right)\cdot\boldsymbol{R'}(\theta)$$

$$\Delta z_{1t}\left(\theta,\alpha_t\right) := \boldsymbol{\Delta z_t}\left(\theta,\alpha_t\right)_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_t}\left(\theta,\alpha_t\right)_1 \qquad z_{2t}\left(\theta,\alpha_t\right) := z_{1t}\left(\theta,\alpha_t\right) + \Delta z_{1t}\left(\theta,\alpha_t\right)$$

$$\Delta z_{1s}(\theta,\alpha) := \boldsymbol{\Delta z_s}(\theta,\alpha)_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_s}(\theta,\alpha)_1 \qquad z_{2s}(\theta,\alpha) := z_{1s}(\theta,\alpha) + \Delta z_{1s}(\theta,\alpha)$$

$$\Delta z_{1t'}\left(\theta,\alpha_t\right) := \boldsymbol{\Delta z_{t'}}\left(\theta,\alpha_t\right)_0 + \mathrm{i}\cdot\boldsymbol{\Delta z_{t'}}\left(\theta,\alpha_t\right)_1 \qquad z_{2t'}\left(\theta,\alpha_{t'}\right) := z_{1t'}\left(\theta,\alpha_{t'}\right) + \Delta z_{1t'}\left(\theta,\alpha_{t'}\right)$$

$$\Delta\xi_{2t}(\theta) := \frac{r_t}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{\alpha_{tP}}^{\pi} \boldsymbol{S_t}\left(\theta,\alpha_t\right)_{0,0} \, d\alpha_t + R'_y(\theta)\cdot\int_{\alpha_{tP}}^{\pi} \boldsymbol{S_t}\left(\theta,\alpha_t\right)_{0,1} \, d\alpha_t\right)$$

$$\Delta\eta_{2t}(\theta) := \frac{r_t}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{\alpha_{tP}}^{\pi} \boldsymbol{S_t}\left(\theta,\alpha_t\right)_{1,0} \, d\alpha_t + R'_y(\theta)\cdot\int_{\alpha_{tP}}^{\pi} \boldsymbol{S_t}\left(\theta,\alpha_t\right)_{1,1} \, d\alpha_t\right)$$

$$\Delta\xi_{2s}(\theta) := \frac{R_0}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{\pi}^{\psi_0+\pi} \boldsymbol{S_s}(\theta,\alpha)_{0,0} \, d\alpha + R'_y(\theta)\cdot\int_{\pi}^{\psi_0+\pi} \boldsymbol{S_s}(\theta,\alpha)_{0,1} \, d\alpha\right)$$

$$\Delta\eta_{2s}(\theta) := \frac{R_0}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{\pi}^{\psi_0+\pi} \boldsymbol{S_s}(\theta,\alpha)_{1,0} \, d\alpha + R'_y(\theta)\cdot\int_{\pi}^{\psi_0+\pi} \boldsymbol{S_s}(\theta,\alpha)_{1,1} \, d\alpha\right)$$

$$\Delta\xi_{2t'}(\theta) := \frac{r_t}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{0}^{2\cdot\beta_0} \boldsymbol{S_{t'}}\left(\theta,\alpha_{t'}\right)_{0,0} \, d\alpha_{t'} + R'_y(\theta)\cdot\int_{0}^{2\cdot\beta_0} \boldsymbol{S_{t'}}\left(\theta,\alpha_{t'}\right)_{0,1} \, d\alpha_{t'}\right)$$

$$\Delta\eta_{2t'}(\theta) := \frac{r_t}{L_t}\cdot\left(R'_x(\theta)\cdot\int_{0}^{2\cdot\beta_0} \boldsymbol{S_{t'}}\left(\theta,\alpha_{t'}\right)_{1,0} \, d\alpha_{t'} + R'_y(\theta)\cdot\int_{0}^{2\cdot\beta_0} \boldsymbol{S_{t'}}\left(\theta,\alpha_{t'}\right)_{1,1} \, d\alpha_{t'}\right)$$

$$\xi_{2s}(\theta) := \xi_1(\theta) + \Delta\xi_{2t}(\theta) + \Delta\xi_{2s}(\theta) + \Delta\xi_{2t'}(\theta)$$

$$\eta_{2s}(\theta) := \eta_1(\theta) + \Delta\eta_{2t}(\theta) + \Delta\eta_{2s}(\theta) + \Delta\eta_{2t'}(\theta) \qquad \xi_{2s}\left(\theta_0\right) = 2.994\times10^{-3}\,mm \qquad \eta_{2s}\left(\theta_0\right) = -3.697\times10^{-3}\,mm$$

Approximations de Haag

Vérification des approximations

$$\sigma 2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0}\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\,d\alpha + \int_{\alpha_{tP}}^{\pi}\left(\left|z_{0t}(\alpha_t)\right|\right)^2\cdot r_t\,d\alpha_t + \int_0^{2\cdot\beta_0}\left(\left|z_{0t'}(\alpha_{t'})\right|\right)^2\cdot r_t\,d\alpha_{t'}\right]$$

$$\kappa_t := \frac{1}{\sigma 2\cdot L_t^2}\cdot\int_{\alpha_{tP}}^{\pi} s_t(\alpha_t)\cdot\left(\left|z_{0t}(\alpha_t)\right|\right)^2\cdot r_t\,d\alpha_t \qquad \kappa_{t'} := \frac{1}{\sigma 2\cdot L_t^2}\cdot\int_0^{2\cdot\beta_0} s_{t'}(\alpha_{t'})\cdot\left(\left|z_{0t'}(\alpha_{t'})\right|\right)^2\cdot r_t\,d\alpha_{t'}$$

$$\kappa_s := \frac{1}{\sigma 2\cdot L_t^2}\cdot\int_{\pi}^{\pi+\psi_0} s_s(\alpha)\cdot\left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\,d\alpha \qquad \kappa := \kappa_t + \kappa_s + \kappa_{t'} \qquad \kappa = 0.5$$

$$\zeta(\theta) := -i\cdot\frac{d}{d\theta}\mathbf{\Delta_1}(\theta) - \kappa\cdot\mathbf{\Delta_1}(\theta) \qquad \zeta(\theta_0) = 2.759\times 10^{-3} - 8.964i\times 10^{-4}\,mm$$

$$\zeta_t(\theta) := \frac{1}{L_t}\cdot\int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t)\cdot e^{i\cdot\theta\cdot\frac{s_t(\alpha_t)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_t(\alpha_t)}{L_t} - \kappa\right)\right]\cdot r_t\,d\alpha_t$$

$$\zeta_s(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot e^{i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_s(\alpha)}{L_t} - \kappa\right)\right]\cdot R_0\,d\alpha$$

$$\zeta_{t'}(\theta) := \frac{1}{L_t}\cdot\int_0^{2\cdot\beta_0} z_{0t'}(\alpha_{t'})\cdot e^{i\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t'}(\alpha_{t'})}{L_t} - \kappa\right)\right]\cdot r_t\,d\alpha_{t'}$$

$$\zeta(\theta) := \zeta_t(\theta) + \zeta_s(\theta) + \zeta_{t'}(\theta) \qquad \zeta(\theta_0) = 2.759\times 10^{-3} - 8.964i\times 10^{-4}\,mm$$

Formule de Haag

$$X_{0t}(\alpha_t) := R_0 - r_t + r_t\cdot\cos(\alpha_t) \qquad Y_{0t}(\alpha_t) := r_t\cdot\sin(\alpha_t) \qquad Z_{0t}(\alpha) := X_{0t}(\alpha) + i\cdot Y_{0t}(\alpha)$$

$$Z_2 := \frac{1}{R_0^3}\cdot\int_0^{2\cdot\beta_0} r_t\cdot\alpha\cdot Z_{0t}(\alpha)\cdot r_t\,d\alpha + 1 \qquad \rho_2 := \left|Z_2\right| \qquad \rho_2 = 1.074 \qquad \varphi_2 := \arg(Z_2) \qquad \varphi_2 = 145.7\,deg$$

$$\mathbf{OA} := R_0\cdot e^{i\cdot\pi} \qquad \zeta_{aPh}(\theta) := \frac{\theta}{2\cdot\psi_0^2}\cdot\rho_2\cdot\mathbf{OA}\cdot\left[-(4\cdot i + \theta)\cdot e^{-i\cdot\varphi_2} + (4\cdot i - \theta)\cdot e^{i\cdot(\psi_0+\varphi_2+\theta)}\right]$$

$$\omega(\theta) := \frac{\psi_0 + \theta}{2} + \varphi_2 \qquad \zeta_{aPh}(\theta) := -\frac{\theta}{\psi_0^2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\cdot\mathbf{OA}\cdot e^{i\cdot\omega(\theta)}\cdot(\theta\cdot\cos(\omega(\theta)) + 4\cdot\sin(\omega(\theta)))$$

$$\zeta_{aPh}(\theta_0) = 1.706\times 10^{-3} - 5.544i\times 10^{-4}\,mm$$

Graphes du déplacement du centre de gravité

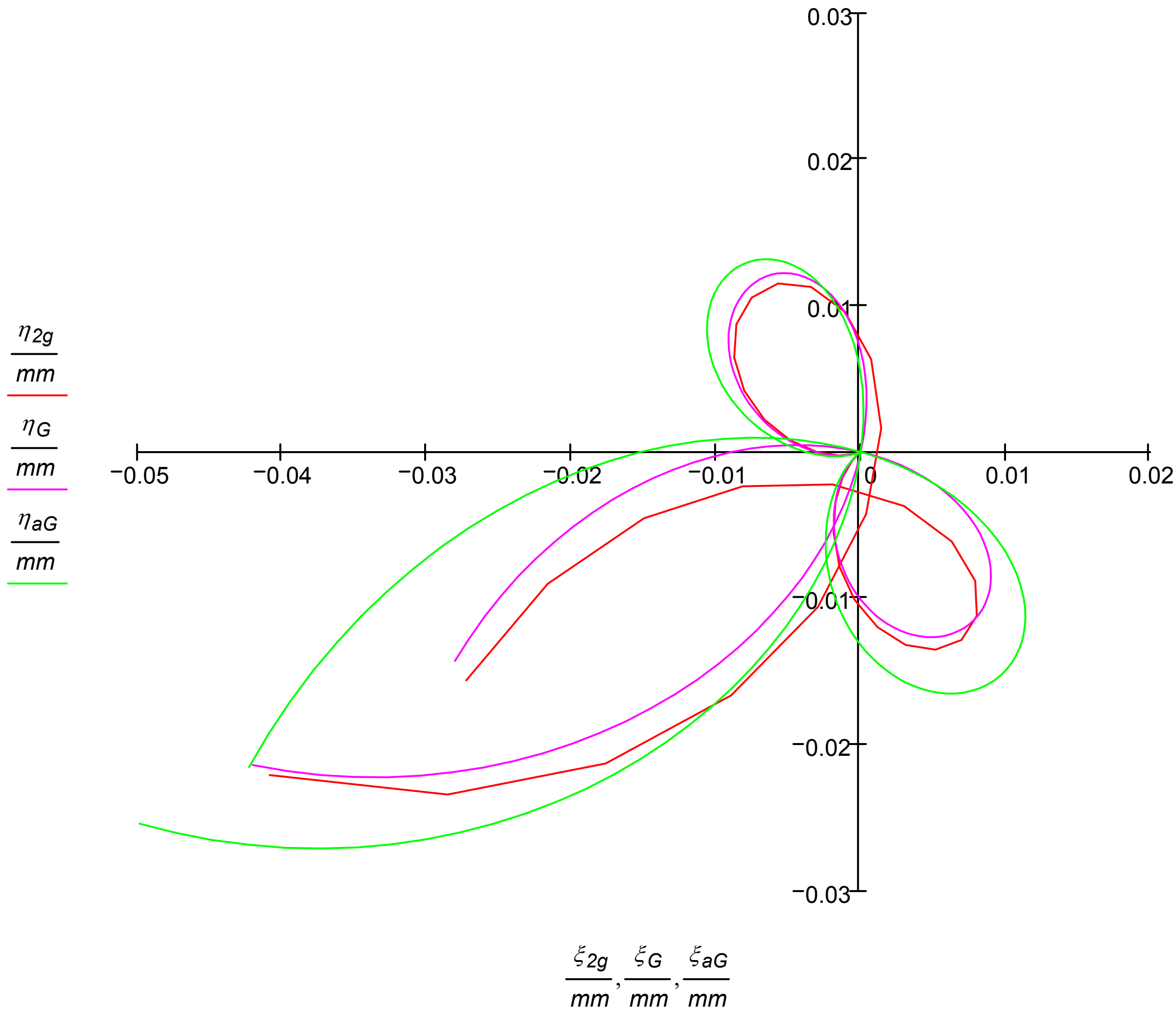
$n := 201$ $\quad i := 0 .. n-1$ $\quad \Delta\theta := \dfrac{4\cdot\pi}{n-1}$ $\quad \theta_i := -2\cdot\pi + i\cdot\Delta\theta$

$m := 41$ $\quad j := 0 .. m-1$ $\quad \Delta\theta_m := \dfrac{4\cdot\pi}{m-1}$ $\quad \theta_{m_j} := -2\cdot\pi + j\cdot\Delta\theta_m$

$\xi_{2g_j} := \xi_{2s}\left(\theta_{m_j}\right)$ $\quad \eta_{2g_j} := \eta_{2s}\left(\theta_{m_j}\right)$ $\quad \xi_{2g_j} := 0$ $\quad \eta_{2g_j} := 0$

Attention: Calcul long !

$\xi_{G_i} := Re\left(\zeta\left(\theta_i\right)\right)$ $\quad \eta_{G_i} := Im\left(\zeta\left(\theta_i\right)\right)$ $\quad \xi_{aG_i} := Re\left(\zeta_{aPh}\left(\theta_i\right)\right)$ $\quad \eta_{aG_i} := Im\left(\zeta_{aPh}\left(\theta_i\right)\right)$



Perturbation de période - spiral non déformé en position de repos

Calcul par intégrations numériques

$\eta(\theta) := Im(\zeta(\theta))$ $\quad Gamma(\theta) := -m_s\cdot g\cdot\dfrac{d}{d\theta}\eta(\theta)$

$\theta(\varphi) := \theta_0\cdot\cos(\varphi)$ $\quad Delta\left(\theta_0\right) := \dfrac{L}{2\cdot\pi\cdot\theta_0\cdot E\cdot I_{33}}\cdot\displaystyle\int_0^{2\cdot\pi} Gamma\left(\theta_0\cdot\cos(\varphi)\right)\cdot\cos(\varphi$ $\quad Delta\left(\theta_0\right) = 2.154\times 10^{-5}$

$$Z_t(\theta_0) := \frac{r_t}{L_t^2} \cdot \int_{\alpha_{tP}}^{\pi} z_{0t}(\alpha_t) \cdot s_t(\alpha_t) \cdot \left[\left(\kappa - \frac{s_t(\alpha_t)}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_t(\alpha_t)}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_t(\alpha_t)}{L_t}\right)\right] d\alpha_t$$

$$Z_s(\theta_0) := \frac{R_0}{L_t^2} \cdot \int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha) \cdot s_s(\alpha) \cdot \left[\left(\kappa - \frac{s_s(\alpha)}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_s(\alpha)}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_s(\alpha)}{L_t}\right)\right] d\alpha$$

$$Z_{t'}(\theta_0) := \frac{r_t}{L_t^2} \cdot \int_{0}^{2\cdot\beta_0} z_{0t'}(\alpha_{t'}) \cdot s_{t'}(\alpha_{t'}) \cdot \left[\left(\kappa - \frac{s_{t'}(\alpha_{t'})}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_{t'}(\alpha_{t'})}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t'}(\alpha_{t'})}{L_t}\right)\right] d\alpha_{t'}$$

$$Z(\theta_0) := Z_t(\theta_0) + Z_s(\theta_0) + Z_{t'}(\theta_0)$$

$$Delta(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z(\theta_0)) \qquad Delta(\theta_0) = 2.154 \times 10^{-5}$$

$$\mu(\theta_0) := -86400 \cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = -1.861} \qquad \boxed{\mu(180 \cdot deg) = 1.441}$$

Approximation de Haag

$$Q(\theta_0) := 5 \cdot J0(\theta_0) - \theta_0 \cdot J1(\theta_0) \qquad \mathbf{OB} := R_0 \cdot e^{i\cdot(\pi+\psi_0)}$$

$$Z_{aPh}(\theta_0) := \frac{-R_0^2}{2 \cdot L_t^2} \cdot \rho_2 \cdot \left(\mathbf{OA} \cdot e^{-i\cdot\varphi_2} + Q(\theta_0) \cdot \mathbf{OB} \cdot e^{i\cdot\varphi_2}\right) \qquad \delta_{aPh}(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_{aPh}(\theta_0))$$

$$\mu_{aPh}(\theta_0) := -86400 \cdot \delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = -1.832} \qquad \boxed{\mu_{aPh}(180 \cdot deg) = 0.806}$$

$$\theta_m := 60 \cdot deg, 65 \cdot deg .. 300 \cdot deg$$